MATH 551 - Problem Set 10

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1.

a) Because we know that the plane is horizontal, we know it can be written as $z = c$ where c is an arbitrary constant. Now, because any vector $x \in H^2$ has it that $\Phi(x, x) = -1$, we know that ${x_1}^2 + {x_2}^2 = {x_3}^3 - 1$. Well, any intersection of a horizontal plane ($z = c$) would intersect with the vectors in H^2 at the constant c, and so therefore the equation that defines that intersection is ${x_1}^2 + {x_2}^2 = c^2 - 1$ and because c is constant, this implies that the intersection is a circle in R^3 with radius $\sqrt{c^2 - 1}$.

b) Borrowing from the argument above, we know for $x \in H^2$ that $x_1 + x_2 - k = -1$ defines the intersection of H^2 with the arbitrary horizontal plane $z = k$. Let's claim the object traced in H^2 by this intersection is a circle with center $y =$ the vertex of H^2, that is $y = (0, 0, 1)$, and examine the distance between y and x. The distance between this proposed center, y, and any point on the intersection, x, in H^2 is defined as $arccosh(-\Phi(x, y)) = arccosh(-(0*x_1)+(0*x_2)+(1*k)) = arccosh(-k)$, which is a constant value. Because x was chosen arbitrarily as a vector in the intersection of H^2 with the horizontal plane, we've shown that the distance between y and any point on the intersection is constant, which implies that the object traced is a circle (in H^2 because we used hyperbolic measures of distance).

2.

a) To show that this curve is in H^2, we must show that $\forall t \; \Phi(\phi(t), \phi(t)) = -1$, so we have $\Phi(\phi(t), \phi(t)) = t^2 + 0 - (t^2 + 1) = t^2 - t^2 - 1 = -1$ and so we are done.

b) A line in H^2 is defined as any non-empty intersection of a plane (that passes through the origin) with H^2. Well, our parameterized curve $\phi(t)$ is 0 everywhere in y, which means that the curve lies exclusively in the xz plane. The intersection of H^2 and the xz plane is non-empty and the xz plane passes through the origin. Therefore, along with our conclusions from part a) we may conclude that $\phi(t)$ is a line in H^2.

c) We begin by taking the derivative of $\phi(t)$ which is $\phi'(t) = (1, 0, \frac{2t}{2\sqrt{t^2+1}}) =$

$(1, 0, \frac{t}{\sqrt{t^2+1}})$. To find the 'speed' at x, all we need is to take $\Phi(\phi'(x), \phi'(x))$. So,

$\Phi(\phi'(-2), \phi'(-2)) = 1 + 0 - \frac{4}{5} = \frac{1}{5}$

$\Phi(\phi'(-1), \phi'(-1)) = 1 + 0 - \frac{1}{2} = \frac{1}{2}$

$\Phi(\phi'(0), \phi'(0)) = 1 + 0 - 0 = 0$

$\Phi(\phi'(1), \phi'(1)) = 1 + 0 - \frac{1}{2} = \frac{1}{2}$

$\Phi(\phi'(2), \phi'(2)) = 1 + 0 - \frac{4}{5} = \frac{1}{5}$